Section 7.6 Solving Radical Equations

The general method for solving equations involving radicals is to isolate the radical on one side of the equation. Raise BOTH sides to the appropriate power to remove the radical and the answer will be on the other side. Thus, if you have a cube root in the problem, you will end up cubing each side to remove the radical.

Example 1

$\sqrt{x} - 3 = 4$ We need to get ONLY the radical on one side so we add 3

$\sqrt{x} = 7$

$x = 49$

Since we "**used the method of squaring**" to solve this equation, we ***must*** show the check. It will not be an unusual occurrence to end up with one or more solutions, some of which are not valid.

$\sqrt{x} - 3 = 4$ Write the *original* equation

$\sqrt{49} - 3 = 4$ substitute

You will be tempted to add 3 to both sides.

During a check, you are to simplify ***each*** **side independently**. You are ***not*** allowed to "add to both sides" or "multiply both sides by …" or any thing else that would affect both sides.

$7 - 3 = 4$

$4 = 4$

This is the proper conclusion of the check.

Example 2

$$\begin{cases} \sqrt{x} + 5 = 2 \\ \sqrt{x} = -3 \\ x = 9 \\ \sqrt{x} + 5 = 2 \\ \sqrt{9} + 5 = 2 \\ 3 + 5 = 2 \\ 8 = 2 \text{ False. No solution} \end{cases}$$

Example 3 $x = \sqrt{x+7} + 5$

We use the method of squaring to remove the radical. In this problem, if we squared both sides, we would have to FOIL the right side. The result would have the middle term *still* with a radical.

Thus: $9 = x + 7 + 10\sqrt{x+7} + 25$ so squaring did not help. We should have isolated the radical on one side before we squared.

$x = \sqrt{x+7} + 5$

$x - 5 = \sqrt{x+7}$ Now we can square both sided and remove the radical

$x^2 - 10x + 25 = x + 7$

$x^2 - 11x + 18 = 0$

$(x-2)(x-9) = 0$

$x = 2 \quad x = 9$ Since we have 2 solutions, we must show 2 checks

$x = \sqrt{x+7} + 5$	$x = \sqrt{x+7} + 5$
$2 = \sqrt{2+7} + 5$	$9 = \sqrt{9+7} + 5$
$2 = 3 + 5$	$9 = 4 + 5$
$2 = 8$ False	$9 = 9$ True

x=9 only solution

Example 4

$$\begin{cases} (2x+1)^{\frac{1}{3}} + 5 = 0 \\ (2x+1)^{\frac{1}{3}} = -5 \\ \left((2x+1)^{\frac{1}{3}}\right)^3 = (-5)^3 \\ 2x + 1 = -125 \\ 2x = -126 \\ x = -63 \end{cases}$$

Raise both sides to the third power (at the step $(2x+1)^{\frac{1}{3}} = -5$)

We do *not* need to show the check because this is "odd numbered root".

Example 5 Pg 468 #46 $\sqrt{6x+7} = 1 + \sqrt{3x+3}$

We are unable to isolate "the" radical. No matter where we move the radicals, we will end up with a "middle term" that involves a radical. Some of the possibilities are more complicated than others. We will need to isolate as best we can and square and then isolate and square once again!

$$\sqrt{6x+7} = 1 + \sqrt{3x+3}$$
$$\left(\sqrt{6x+7}\right)^2 = \left(1 + \sqrt{3x+3}\right)^2$$
$$6x + 7 = 1 + 2\sqrt{3x+3} + 3x + 3$$
$$6x + 7 = 2\sqrt{3x+3} + 3x + 4$$
$$3x + 3 = 2\sqrt{3x+3}$$
$$\left(3x+3\right)^2 = \left(2\sqrt{3x+3}\right)^2$$
$$9x^2 + 18x + 9 = 4\left(3x+3\right)$$
$$9x^2 + 18x + 9 = 12x + 12$$
$$9x^2 + 6x - 3 = 0$$
$$3\left(3x^2 + 2x - 1\right) = 0$$
$$3\left(3x-1\right)\left(x+1\right) = 0$$
$$x = \frac{1}{3} \quad x = -1$$

$$\sqrt{6x+7} = 1 + \sqrt{3x+3}$$
$$\sqrt{6 \cdot \frac{1}{3} + 7} = 1 + \sqrt{3 \cdot \frac{1}{3} + 3}$$
$$\sqrt{2+7} = 1 + \sqrt{1+3}$$
$$3 = 1 + 2$$

True.

$$\sqrt{6x+7} = 1 + \sqrt{3x+3}$$
$$\sqrt{6(-1)+7} = 1 + \sqrt{3(-1)+3}$$
$$\sqrt{-6+7} = 1 + \sqrt{-3+3}$$
$$1 = 1$$

True

Solution: $x \in \left\{\frac{1}{3}, -1\right\}$

Section 7.8 Complex Numbers

We define $\sqrt{-1} = i$ The letter i is used to represent "imaginary" numbers.

$$\left(\sqrt{-1}\right)^2 = i^2 = -1$$

So $i^3 = i \cdot i^2 = -1i$ ie. $-i$

$$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

$$i^5 = i \cdot i^4 = i$$

And so on.

When real numbers are combined with imaginary numbers, we get “complex numbers”

Complex numbers are normally written in the form $a+bi$
Where a is the “real part” of the complex number and b is the “imaginary part” of the complex number.

Complex numbers follow most of the same rules as do real numbers. The one rule not followed has to do with inequality.

This “i” in $a+bi$ is treated much like a variable when it comes to the normal algebraic operations:

$$\begin{array}{rcr} 3 & + & 5i \\ -4 & - & 7i \\ \hline -1 & - & 2i \end{array}$$ as you would expect.

$$(3+5i)(-4-7i)$$

$$\begin{array}{lll} -12 & -21i & \\ & -20i & -35i^2 \\ 35 & & \\ \hline 23 & -41i & \end{array}$$

We use FOIL. Notice i^2 is – 1 so the $-35i^2$ becomes 35.

$$\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1} = 4i$$

$$\sqrt{-5} \cdot \sqrt{-7} = i\sqrt{5} \cdot i\sqrt{7} = i^2\sqrt{35} = -\sqrt{35}$$

NOTE: We do NOT do this: $\sqrt{-5} \cdot \sqrt{-7} = \sqrt{(-5)(-7)} = \sqrt{35}$

Like when working with radicals before, we do NOT leave complex numbers in the denominator of a fraction.

$\frac{-5+9i}{1-2i}$ We multiply by the "conjugate" . That is we multiply by what will remove the complex number from the bottom. That method involves "difference of squares".

$\frac{-5+9i}{1-2i} \cdot \frac{1+2i}{1+2i}$. Use FOIL to clear each and get

$\frac{-23-i}{5}$ We want the final answer in the proper form of $a+bi$ so

we end up with $-\frac{23}{5}-\frac{1}{5}i$

NOTE: We always write final answers that are complex numbers in the form of $a+bi$!